The approaches for measurement uncertainties evaluation

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Different methods and tools are now available for measurement uncertainty evaluation. These new methods comply with the concepts and recommendations of Guide to the expression of uncertainty in measurement (GUM). During the presentation, the author will introduce several alternatives for laboratories and notably those based on inter-laboratory comparisons : modelling approach, single laboratory validation approach, inter-laboratory validation approach and proficiency testing approach. The new approaches for uncertainty evaluation will certainly streamline the measuring results or test reports, their comparability and their traceability to SI units.

Plusieurs outils et méthodes sont actuellement disponibles pour évaluer l'incertitude de mesure. Ces nouvelles méthodes obéissent essentiellement aux concepts et aux recommandations du GUM (Guide to the expression of Uncertainty in Measurement).
Dans cet article, l'auteur présente plusieurs solutions d'évaluation à l'attention des laboratoires et notamment celles concernant les comparaisons entre laboratoires. Il s'agit des approches par modélisation, par validation dun seul laboratoire, par validation inter-laboratoires et enfin de la démarche des tests d'aptitude. Ces nouvelles démarches d'évaluation de l'incertitude permettront sûrement de rationaliser les résultats de mesure ou les rapports d'essais, ainsi que leur comparabilité et leur traçabilité aux unités SI.

Many important decisions are based on measurement or test results : the results can be used for example to assess a noise level, to check material against specification or statutory limits…Whenever decisions are based on measurement, it is important to have some indication on the quality of the results. The uncertainty of measurement is the simplest way to express the reliability of the result.
Since the standard ISO / IEC 17025 "General requirements for the competence of testing and calibration laboratories" has been published in 2000, the concept of uncertainty of measurements or test results seems to be a key issue for laboratories and their clients. Up to now, only calibration laboratories were deeply involved in uncertainty evaluation. The concept of uncertainty is certainly difficult to understand and to admit by people not familiar with technical activities. It is now recognized in the testing community that it is as important to communicate the uncertainty related to a specific measurement as it is to report the measurement result itself. It is also clear that a measurement or a test result without an assessment of its reliability is completely useless. Without knowing uncertainty it is impossible to assess the comparison between different measurements with the same parameter or to compare a result with a specification limit. Measurement and test results represent the basic information for the conformity statement of many products or activities.
The evaluation of uncertainty is often regarded as a difficult and arduous task. The Guide to the Expression of Uncertainty in Measurement (GUM) provides international agreed concepts, recommendations and a procedure for the evaluation of uncertainty. consistency with the GUM requires the basic concept to be accepted and the recommendation to be followed.

The conference will present the three levels of reading of the GUM: Concepts, recommendations and a procedure. The different methods and tools for the evaluation of uncertainty components will also be introduced: modelling approach, single laboratory validation approach, inter-laboratory validation approach and proficiency testing approach. These presentation will be based on the new International or European document EA 4/16 EA Guidelines on the expression of uncertainty in quantitative testing and ISO / TS 21748 Guidance for the use of repeatability, reproducibility and trueness estimates in measurement uncertainty estimation.

The three levels of reading of the GUM

The concept of uncertainty and principles are defined in the GUM (Guide for the Expression of Uncertainty in Measurement). GUM [1] is based on sound theory and provides a consistent and transferable evaluation of measurement uncertainty.

Three levels in the GUM can be identified:

- Basic concepts
- Recommendations
- Evaluation procedure

Consistency requires to approve the basic concept and to follow recommendations. The procedure proposed in the GUM (see chapter 8 of GUM: Summary of procedure for evaluating and expressing uncertainty) is one of the practical approaches for evaluating uncertainty but different others approaches, all fully compliant with the GUM will be presented in this document. In some circumstances for example when it is impractical to identify all major individual measurement uncertainty contributions, only methods based on repeatability, reproducibility and trueness estimate could be used.

Basic concepts of the GUM

The basic concepts in uncertainty evaluation are:

- The knowledge about any quantity that influence the measurand is in principle incomplete and can be expressed by a probability density function (PDF) for the values attributable to the quantity base on that knowledge.
- The expectation value of that PDF is taken as the best estimate of the value of the quantity.
- The standard deviation of that PDF is taken as the standard uncertainty associated with that estimate.

The PDF is based on knowledge about a quantity that may be inferred from.

- Repeated measurement: type A evaluation
- Scientific judgement based on all the available information on the possible variability of the quantity: type B evaluation

Recommendations

The GUM provides the following recommendations:

A model formulated to account for the interrelation of the input quantities that influence the measurand.
This is certainly the most important aspect of the GUM, we need to establish a relationship between the measurand Y and N others quantities $X_1, X_2, \ldots, X_N$ (input quantities) through a function f:

$$Y = f(X_1, X_2, \ldots, X_N)$$

where $X_1, X_2, \ldots, X_N$ are all the relevant information used to calculate the measuring result. Since this is generally the most difficult part of the evaluation, the use of a cause-effect relationship linking the input quantities to the measurand is recommended. Correction included in the model to account for systematic effects; such corrections are essential for achieving traceability to stated references (e.g. CRMs, reference measurement procedure, SI units).
Measurement result reports which indicate the value and a quantitative indication of the quality of that result (the uncertainty).

The provision of an interval about the measurement result that may be encompass a large fraction of the values that could be reasonably attributed to the measurand.

Procedure for uncertainty evaluation

People have often reduced the GUM to the procedure summarized in the Chapter 8 of this document. The following steps are proposed in the procedure.

- Derivation of the model of the measurement (the most difficult task). The provision of probability density function (PDFs) for the input quantities to the model, given information about these quantities.

- In many case in practice, it is necessary to specify only the expectation value and standard deviation of each PDF. i.e. the best estimate of each quantity and the standard uncertainty associated with that estimate.

- Propagation of uncertainty. The basic procedure (the law of propagation of uncertainty) can be applied to linear or linearized models, but subject to some restrictions.

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

This law, called propagation law of uncertainty is based on a first order Taylor series expansion where,

u_c^2 (y) is the variance on the measuring result

$\frac{\partial f}{\partial x_i}$ is the partial derivative

$u(x_i, x_j)$ is the covariance between x_i and x_j

- Stating the complete result of a measurement by providing the best estimate of the value of the measurand, the combined standard uncertainty associated with that estimate and an expanded uncertainty.
Some evolutions will appear soon, a supplement 1 to the GUM [2] will develop advices concerning the numerical methods for the propagation of distribution.

In the procedure, presented in the GUM, it is proposed to the propagation law of uncertainty. The model has mutually independent input quantities $X=(X_1,X_2,X_3)^T$ whose value is estimated by x_i with associated standard uncertainty $u(x_i)$. The value of the output quantity Y is estimated by y, with associated standard uncertainty u(y).

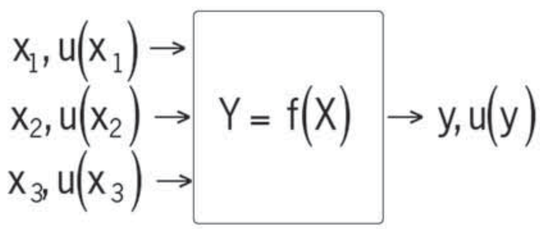

Fig. 1 : Illustration of the law of propagation of uncertainty

In the GUM supplement 1 it is proposed to propagate distribution instead of the propagation of variance as it is described in the GUM procedure of chapter 8.

The figure 2 show an illustration of the concept of propagation of distribution.

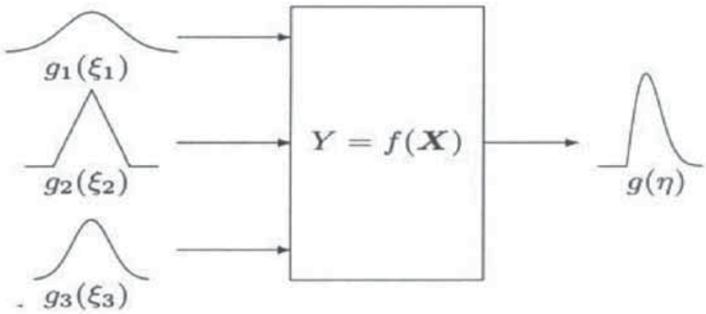

Fig. 2: Illustration of the propagation of distributions

The model input quantities are $X=(X_1,X_2,X_3)^T$.

The probability density function , for X_i,i=1,2,3, are Gaussian, triangular and Gaussian respectively.

The probability density function g(η) for the value of the output quantity *Y* is indicated as being asymmetric, as can arise for non linear models. An asymmetric output PDF can also arise when the PDF for the values of input quantities are asymmetric.

Approaches to uncertainty estimation

The ISO / IEC 17025 "General requirements for competence of testing and calibration laboratories" in its paragraphed dealing with uncertainty of measurement (§ 5.4.6) referred at the same level the ISO 5725 "Accuracy (trueness and precision) of measurement methods and results" and the GUM. So it seems important for testing laboratory to develop approaches consistent with the concept and recommendations of the GUM but using others tools than the procedure of chapter 8.

The paragraph 5.4.6.2 of ISO / IEC 17025 justifies entirely these new approaches : *"Testing laboratories shall have and shall apply procedures for estimating uncertainty of measurement. In certain cases the nature of the test method may preclude rigorous, metrologically and statistically valid, calculation of uncertainty of measurement. In these cases the laboratory shall at least attempt to identify all the components of uncertainty and make a reasonable estimation of the uncertainty. Reasonable estimation shall be based on knowledge of the performance of the method and on the measurement scope and shall make use of, previous experience and validation data, for example".*

Road map of the different approaches

The figure 3 below shows a road map for uncertainty evaluation.

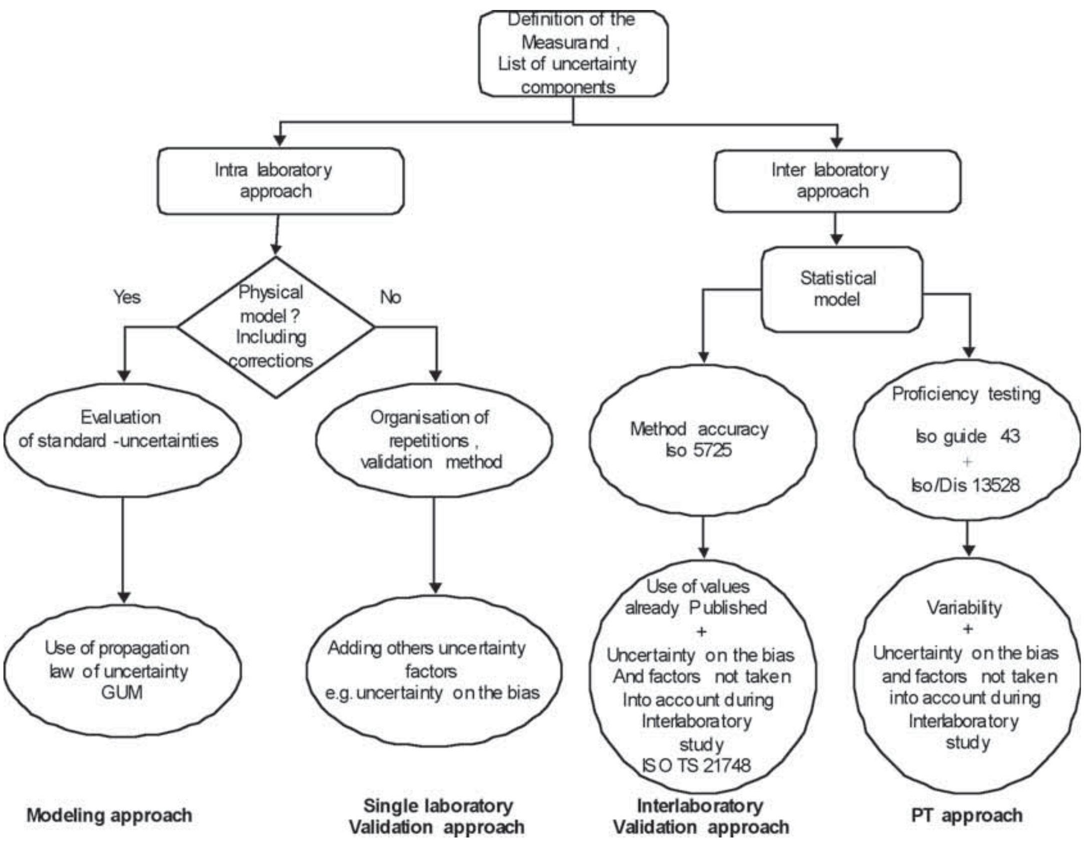

Fig. 3 : Typology of uncertainty evaluation methods

This classification is based on distinction between uncertainty evaluation carried out by the laboratory itself (called intra-laboratory approach) and uncertainty evaluation based on collaborative studies (called inter-laboratory approach). These approaches have been presented in several documents [3], [4].

Both two approaches are then subdivided into:

- Modelling of the measurement process and use of the propagation law of uncertainty and single laboratory validation approach,
- Use of performance method data (ISO 5725) [5] and Proficiency testing data (ISO-Guide 43) [6] for inter-laboratory approaches.

Common points between the different approaches

Whatever the methods intended to be used, it is always important:

- To define clearly, with no ambiguity the measurand or the characteristic to be measured, analysed or tested.
- To analyse carefully the measuring or testing process in order to identify the major components of uncertainty and to examine if they are taken on board in the application of the law of uncertainty or if they are active during the repetition of observations organised to evaluate repeatability and reproducibility or if they are included in collaborative studies. It is also important to admit that in some situations, we are not in a position to identify the components of the uncertainty: the symptom of this can be seen, when the uncertainty evaluated by applying the GUM procedure leads to a value of much less large uncertainty than the variation observed among different laboratory intercomparisons.

Where sampling activities are performed, it is also important to define clearly the measurand. For example, do we seek information related to the sample transmitted to the laboratory for test or do we need information concerning the batch from which is prepared the sample ? It is obvious that the uncertainty will be different in both cases.

Presentation of the four approaches

The "modelling approach"

The GUM procedure of evaluation of uncertainty is described in chapter 8 of the GUM. This procedure is based on a model (mathematical model) formulated to account for the interrelation of the input quantities that influence the measurand. Correction is included in the model to account for systematic effects; such corrections are essential to achieve Traceability to stated references (e.g. CRMs, reference measurement procedure, SI units). The application of the propagation law of uncertainty enables to evaluate the combined uncertainty on the result. The model is often expressed in the form of an equation called "mathematical model" in the GUM.

The "single laboratory validation approach"

When the modelization of the measuring process may be infeasible for economic or other reasons. In such case alternative approaches may be used. In particular, the major sources of variability can often be assessed by

validation method study. Repeatability and reproducibility can be obtained by organising experimental work inside the laboratory. Information can also be obtained from quality control data (control chart).

The first difficulty is to include during the repetition of the experiment a majority of influence quantities that can affect the result.

The second difficulty is to assess the bias (or trueness of the method). The use of CRMs, comparison with definitive or reference method can be a solution to evaluate the component of uncertainty related to the trueness.

The "interlaboratory validation approach"

When the model describing the measuring process is not available, for any reasons, the major sources of variability can often be assessed by inter-laboratory studies stated in ISO 5725 "Accuracy (trueness and precision) of measurement methods and results" which provide estimates of repeatability (repeatability standard deviation s_r), reproducibility (reproducibility standard deviation s_R) and sometimes trueness of the method (measured as a bias with respect to a known reference value). This approach is fully described in ISO / TS 21748 [7] "Guidance for the use of repeatability, reproducibility and trueness estimates in measurement uncertainty estimation".

The "PT approach"

Proficiency Test (PT) are intended to check periodically the overall performance of a laboratory. The laboratory's results from its participation in proficiency testing can accordingly be used to check the evaluated uncertainty, since that uncertainty should be compatible with the spread of results obtained by that laboratory over a number or proficiency test rounds.

The "PT approach" can also be used to evaluate the uncertainty. If the same method was used by all the participants to the PT scheme, then the standard deviation extract from individual results could be considered as a preliminary evaluation of the measurement uncertainty.

For an individual laboratory, the bias and its uncertainty combined with the within laboratory reproducibility may be used as an estimate of the measurement uncertainty.

Combination of different approaches

In fact, very often a combination of the different approaches needs to be used to assess the uncertainty. For example when a laboratory decides to use the modelling approach, the repeatability of the measuring process can be assessed by using a quality control chart which provides a good estimate of the within laboratory reproducibility. The use of inter-laboratory validation approach can require the application (by the CRM supplier) of the modelling approach to evaluate the uncertainty on the reference value of the CRM used to estimate the trueness of the method.
If the measurand includes sampling, then mixing methods for evaluating uncertainty due to sampling and test will be appropriate.

Conclusion

In a recent past, testing laboratories were arguing against the difficulties for the evaluation of uncertainties. A lot of them were reluctant to the application of the propagation law of uncertainty and its apparent mathematical complexity.

The different approaches presented in this paper show several possibilities for laboratories and particularly those based on inter laboratory comparisons.

But the difficulties remain the same, whatever the method used to evaluate the uncertainty of the result. It is essential to define clearly the measurand and to analyse carefully the measuring process in order to identify the factors which influence the result. These two tasks requires more technical competencies in measurement techniques than mathematical skills.

The new approaches for uncertainty evaluation will certainly bring facilities in the presentation of results, their comparability and their traceability to SI Units.

Bibliography

[1] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML - Guide to the expression of uncertainty in measurement, ISO 1995

[2] JCGM, Guide to the expression of uncertainty in measurement supplement 1- numerical methods for the propagation of distributions, to be published

[3] Christophe PERRUCHET et Marc PRIEL, Estimer l'incertitude – Mesures & Essais, AFNOR, 2000 Spanish version published by AENOR 2000

[4] EA – 4/16 – Guidelines on the expression of uncertainty in quantitative testing, European co-operation of accreditation, 2004 French and English version available on the LNE web site www.lne.fr

[5] ISO 5725, Accuracy (trueness and precision) of measurement methods and results, ISO 1994

[6] ISO/IEC 43-1 Proficiency testing by interlaboratory comparisons – Part 1 Development and operation of proficiency testing schemes, ISO 1997

[7] ISO / TS 21748 – Guidance for the use of repeatability, reproducibility and trueness estimates in measurement uncertainty estimation, ISO 2004

[8] Eurolab Technical Report N° 1/2002 June 2002 "Measurement Uncertainty in Testing" EUROLAB 2002